3.5
16)

$$f(x) = \cos^2(3\sqrt{x}) = [\cos(3\sqrt{x})]^2$$

template

$$x \mapsto 3\sqrt{x} \mapsto \cos x \mapsto (\ )^2$$

$f(x) = x^2$ $\quad f'(x) = 2x$

$g(x) = \cos(3\sqrt{x})$ $\quad g'(x) = -\sin(3x^{\frac{1}{2}}) \cdot \frac{3}{2\sqrt{x}}$

Rule
$[f(g(x))]' = f'(g(x)) \cdot g'(x)$

$f(x) = \cos(x)$ $\quad f'(x) = -\sin(x)$

$g(x) = 3\sqrt{x}$
$= 3x^{1/2}$ $\quad g'(x) = 3\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$
$= \frac{3}{2\sqrt{x}}$

$$\frac{dy}{dx} = 2(\cos(3\sqrt{x})) \cdot \left(-\sin(3\sqrt{x}) \cdot \frac{3}{2\sqrt{x}}\right)$$

22) $y = \cos^3\left(\frac{x}{x+1}\right)$

$$x \mapsto \frac{x}{x+1} \mapsto \cos(\quad) \mapsto (\quad)^3$$

$f(x) = x^3 \qquad f'(x) = 3x^2$

$g(x) = \cos x \qquad g'(x) = -\sin x$

$h(x) = \frac{x}{x+1} \qquad h'(x) = \frac{(1)(x+1)-(x)(1)}{(x+1)^2} = \frac{1}{(x+1)^2}$

$$\left(f(g(h(x)))\right)' = f'(g(h(x)))\cdot\left(g(h(x))\right)'$$

$$= f'(g(h(x)))\cdot g'(h(x))\cdot h'(x)$$

$$= 3\left(\cos\left(\frac{x}{x+1}\right)\right)^2\cdot\left(-\sin\left(\frac{x}{x+1}\right)\right)\left(\frac{1}{(x+1)^2}\right)$$

Algebra "technique"

$$\frac{x}{x+1} = \frac{(x+1)-1}{(x+1)}$$

$$= \left(\frac{x+1}{x+1}\right) - \frac{1}{x+1}$$

$$= 1 - \frac{1}{x+1}$$

~~Shazam~~-Jon

$$\frac{d}{dx}\left(-(x+1)^{-1}\right)$$

$$= +(x+1)^{-2}(1)$$

17) $y = 4\cos^5(x) = 4[\cos(x)]^5$

$$\frac{dy}{dx} = 4\frac{d}{dx}\left([\cos(x)]^5\right) =$$

$$4\left(5(\cos(x))^4(-\sin x)\right)$$

$$= -20(\sin x)(\cos x)^4$$

26) $$\left[x^4 - \sec(4x^2-2)\right]^{-4}$$

$$x \mapsto 4x^2-2 \mapsto x^4 - \sec(x) \mapsto x^{-4}$$

$f(x) = x^{-4}$ $\quad$ $f'(x) = -4x^{-5}$

$g(x) = x^4 - \sec(x)$ $\quad$ $g'(x) = 4x^3 - \sec(x)\tan(x)$

$h(x) = 4x^2 - 2$ $\quad$ $h'(x) = 4(2x)$

$$\underline{f'(g(h(x)))} \cdot \underline{g'(h(x))} \cdot \underline{h'(x)}$$

$$\underline{-4(x^4 - \sec(4x^2-2))^{-5}} \cdot \underline{4(4x^2-2)^3 - \sec(4x^2-2)\tan(4x^2-2)} \cdot \underline{4(2x)}$$

3.6 / Ex 1

find $\frac{dy}{dx}$ if $5y^2 + \sin y = x^2$

think "y is a $f^{\underline{n}}$ of x"

$$\frac{d}{dx}\left(5y^2 + \sin y = x^2\right)$$

$$\Rightarrow \frac{d}{dx}(5y^2) + \frac{d}{dx}(\sin y) = \frac{d}{dx}(x^2)$$

$$10y^1 \cdot \frac{dy}{dx} + \cos(y)\left(\frac{dy}{dx}\right) = 2x$$

$$\frac{dy}{dx}(10y + \cos y) = 2x$$

$$\frac{dy}{dx} = \frac{2x}{10y + \cos y}$$

$$\frac{d^2y}{dx^2}$$

$$y = f(x, z)$$

$$\frac{\partial y}{\partial x}, \frac{\partial y}{\partial z}$$

$$\frac{\partial^2 y}{\partial x^2}, \frac{\partial^2 y}{\partial x \partial z} \ldots$$

3.5/22

$$y = \cos^3\left(\frac{x}{x+1}\right) = \left[\cos\left(\frac{x}{x+1}\right)\right]^3$$

$$x \longmapsto \underset{h}{\frac{x}{x+1}} \longrightarrow \underset{g}{\cos(\quad)} \longrightarrow \underset{f}{(\quad)^3}$$

$f(x) = x^3$ $\qquad f'(x) = 3x^2$

$g(x) = \cos(x)$ $\qquad g'(x) = -\sin(x)$

$h(x) = \frac{x}{x+1}$ $\qquad h'(x) = \frac{1}{(x+1)^2}$

$$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{(1)(x+1) - (x)(1)}{(x+1)^2} = \frac{1}{(x+1)^2}$$

Rule

$$\frac{d}{dx} f(g(h(x))) = f'(g(h(x))) \cdot \frac{d}{dx}(g(h(x)))$$

$$= f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$$

$$\frac{d}{dx}(g(h(x))) = g'(h(x)) \cdot h'(x)$$

combining everything

$$\underbrace{3\left(\cos\left(\frac{x}{x+1}\right)\right)^2}\underbrace{\left(-\sin\left(\frac{x}{x+1}\right)\right)}\underbrace{\left(\frac{1}{(x+1)^2}\right)}$$

The Moral

Everytime I use the chain rule, I multiply by another factor.

24) $\sqrt{3x-\sin^2(4x)} = (3x-[\sin(4x)]^2)^{1/2}$

$$x \mapsto \underset{i}{4x} \to \underset{h}{\sin[\ \ ]} \to 3x - \underset{g}{[\ \ ]^2} \to \underset{f}{(\ \ )^{1/2}}$$

$f(x) = x^{1/2}$

$g(x)^* = (3x)^* - x^2$ not the same x

$h(x) = \sin(x)$

$i(x) = 4x$

chk $\cancel{f(g(h(i(x)))) = \sqrt{3x - (\sin(4x))^2}}$

really $f(3x - g(h(i(x))))$

24) $\sqrt{3x-\sin^2(4x)} = \left(3x-[\sin(4x)]^2\right)^{1/2}$

$$x \longmapsto \underset{i}{4x} \to \underset{h}{\sin[\ ]} \to 3x - \underset{g}{[\ ]^2} \to \underset{f}{(\ )^{1/2}}$$

$f(x) = x^{1/2} \quad \longrightarrow \quad f'(x) = \frac{1}{2\sqrt{x}}$

$g(x)^* = x^2 \quad \longrightarrow \quad g'(x) = 2x$

$h(x) = \sin(x) \qquad h'(x) = \cos x$

$i(x) = 4x \qquad i'(x) = 4$

$$\frac{d}{dx}\Big(f\big(3x - g(h(i(x)))\big)\Big)$$

$$= f'(\sim)\cdot\left(3 - \frac{d}{dx}\big(g(h(i(x)))\big)\right)$$

$$= f'(\sim)\cdot\left[3 - g'(h(i(x)))\cdot h'(i(x))\cdot i'(x)\right]$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{3x-\sin^2(4x)}}\left[3 - 2(\sin(4x))^1\cdot\cos(4x)\cdot 4\right]$$

25)

$$\left[x+\csc(x^3+3)\right]^{-3}$$

$f(x)=x^{-3}$ $g(x)_1=x+\csc(x^3+3)$

$f'(u)=-3x^{-4}$ $g'(x)_1=1+\csc$

$g(x)_2=x+\csc x$ $h(x)=x^3+3$

$g'(x)_2=1+\csc x\cot x$ $h'(x)=3x^2$

$1+\csc(x^3+3)\cot(x^3+3)\cdot 3x^2$

$$-3\left(\left(x+\csc(x^3+3)\right)\cdot\left(1+\left(\csc(x^3+3)\cot(x^3+3)\right)\cdot 3x^2\right)\right)$$

3.6) find $\frac{dy}{dx}$ if $5y^2+\sin y=x^2$

'think of y as a f^n of x'

$$\frac{d}{dx}\left(5y^2+\sin y=x^2\right)$$

$$\frac{d}{dx}(5y^2)+\frac{d}{dx}(\sin y)=\frac{d}{dx}(x^2)$$

$$10y\cdot\frac{dy}{dx}+\cos(y)\cdot\frac{dy}{dx}=2x$$

$$\frac{dy}{dx}(10y+\cos y)=2x$$

$$\frac{dy}{dx}=\frac{2x}{10y+\cos y}$$